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Understanding the Standard Model

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Freedom from Adler-Bell-Jackiw anomalies is a primary requirement for the renormalizability of a gauge theory of chiral fermions, which forms the basis of the successful standard model of electroweak interactions and its many extensions. In this article, we explore to what extent, the assumptions behind the standard model as well as the observed quantization of electric charges of quarks and leptons can be understood using the various anomaly constraints and how the situation changes as one tries to incorporate a non-vanishing neutrino mass .

[†] Dedicated to the memory of Robert E. Marshak, to be published in the memorial volume by George Sudarshan.

I. Introduction:

Robert E. Marshak will be remembered for his many contributions to our understanding of particle physics such as the V-A theory of weak interactions with George Sudarshan, the two meson hypothesis with Hans Bethe etc. These ideas are part of the history of particle physics and will be remembered far beyond the time when our generation of physicists are gone. Those of us, who were lucky enough to have been associated with him and to have shared his enthusiasm for physics and life, will always remember him not only for his deep intuition into physics but also for his generosity of spirit, for being a constant voice of reason and for his unflagging commitment to better the world in every way he could.

My association with Bob Marshak dates back some twentyfive years, starting first as a student at Rochester, then as a colleague and collaborator at CCNY and Virginia Polytechnic Institute at Blacksburg. During this period, we bounced many ideas off each other, discarded most but advanced a few in print. I want to take this opportunity to briefly recall one idea that we particularly felt very strongly about. It has to do with the suggestion that physics beyond the standard model must contain local $B - L$ as a symmetry of nature[1]. This idea originated in 1979 and 1980 [1] from the strong prejudice that the standard model must be extended to include the right-handed neutrino to make it completely quark-lepton symmetric. Marshak was of course one of the inventors, along with Gamba and Okubo, of the idea of Baryon-Lepton symmetry [2] which in the modern language becomes quark-Lepton symmetry and the idea had already been successfully used

by Bjorken and Glashow[3] to predict the existence of the charm quark. Extending the same quark lepton symmetry to the standard model leads naturally to a left-right symmetric (LRS) model of weak interaction , where the $U(1)$ symmetry is nothing but the $B - L$ generator.(Although the LRS model had already been discussed by this author in collaboration with Pati, Salam and Senjanović[4], the identification of the $U(1)$ generator with $B - L$ was not known). In the standard model, due to the absence of the right-handed neutrino, the $B - L$ is an anomaly-free global symmetry but since $Tr[B - L]^3 \neq 0$, this symmetry cannot be gauged; however, in the presence of the ν_R , the cubic $B - L$ anomaly vanishes making $B - L$ a gaugeable symmetry. Now, if we "subtract" $B - L$ from the weak hypercharge, the "left-over" piece can be identified with the right-handed weak isospin I_{3R} and it then becomes possible to recast the electric charge formula in a more physical form[1]

$$Q = I_{3L} + I_{3R} + \frac{B - L}{2} \quad (1)$$

Once I_{3R} emerges as a gauge symmetry, it is natural to gauge the entire right-handed gauge symmetry present among the fermions and of course this leads to the LRS models. The formula in equation 1 has several interesting consequences such as A) the neutrino being a Majorana particle with the smallness of its mass being connected to the scale of spontaneous parity violation[5], B) the possible existence of a new baryon violating process , the neutron-anti-neutron oscillation[6] etc. which under certain circumstances may be observable. Heroic experiments were carried out at Grenoble[7] to

search for the neutron-anti-neutron oscillation; no evidence for this process were found at the level of their precision; however it yielded the best upper limit on the process. On the other hand, the idea of a Majorana neutrino is now-a-days an integral part of the discussion in grand unified theories as well as neutrino phenomenology. The experimental results in this front may be more hopeful. If recent indications of possible non-vanishing neutrino mass in several experiments such as the solar neutrino observations as well as the atmospheric neutrino data is confirmed in the future, a spontaneously broken local $B - L$ symmetry as an integral part of physics beyond the standard model will be confirmed. Further possibilities for unification that can incorporate the $B - L$ symmetry, such as SO(10) grandunification, , and preon models etc, were discussed during the past decade and have been summarized in the excellent book[8] by Marshak, finished only a few days prior to his death. I will not discuss these ideas any further in this article. Instead I want to focus on another of Bob Marshak's obsessions during the last five years of his life, having to do with a better understanding of the origin of the immensely successful standard model[9].

In late eighties, Marshak along with C. Q. Geng[10] began a program of trying to understand the assumptions behind the standard model starting from the requirement of anomaly cancellation[11]. Their work was subsequently followed up by a number of authors including this one[12, 13, 14, 15]. These works have led to several remarkable results that throw light on (and even clarify) some of the basic assumptions on which the standard model rests. In this article, I will discuss the present status of this approach.

II. Statement of the problem:

In discussing the standard model[9], one usually postulates the following:

- a) the known set of fermions ($u^\alpha, d^\alpha, e, \nu$) for each generation (where the superscript $\alpha = 1, 2, 3$ denotes the color index);
- b) the local gauge symmetry of nature is $SU(3)_c \times SU(2)_L \times U(1)_Y$;
- c) and the gauge quantum numbers for the fermions are so chosen as to match their observed properties e.g. the left-handed up and down quarks are part of an $SU(2)_L$ doublet (as are the (ν_L, e_L)) to produce the correct beta decay interactions; their Y quantum numbers are so chosen as to reproduce the correct values of the electric charges etc.

In view of the extraordinary success of this model, it is interesting to ask whether it is possible to reproduce these assumptions starting from a more economical principle.

One may consider two approaches to this problem; in the first one to be called the top-down approach, one may start from some general principles (perhaps String theory , Higher dimensional theory etc), and derive a theory of forces and matter at a high scale such as the Planck scale and as this theory is reduced to represent physics at lower energies, the standard model may emerge. The heterotic string theory of Gross, Harvey, Martinec and Rohm[16] could be such a theory- but as is well known, it does not lead to a unique theory at low energies . If eventually it does, then it would provide an immensely satisfactory basis for the standard model and practically, pre-empt any other approach. A less ambitious point of view in the same top-down philosophy, is to start with a grandunified theory; but as is well-known, the

nature of the low energy physics in the grand unified theories depends a great deal on the choice of Higgs multiplets that are used to break the model down to the standard model. We will briefly remark on this in a subsequent section.

Another approach, which could be called the "bottom-up" approach, is to proceed from the low energy side, using again some general principle and see how much of the assumptions behind the standard model can be understood. It is this approach, that we will discuss in this article. It is perhaps fair to point out that the top-down approach has one advantage over the bottom-up philosophy i.e. the former can generate detailed dynamics whereas the latter has so far only been used to shed light on the "geometrical" properties such as quantum numbers and gauge groups etc.

III. Standard model quantum numbers from Anomalies:

Our basic tool will be the freedom from Adler-Bell-Jackiw anomalies[13] required for the consistent renormalization and unitarity of a gauge field theory. I will not follow the historical path ; rather, will follow the discussion of a recent paper by Frampton and I[17] and gradually embed the earlier discussions where they fit in. It is clear that anomalies alone cannot lead to a definite theory without some way to specify the underlying chiral fermions and/or some knowledge of the gauge symmetry that is responsible for the dynamics. Since we are exploring the nature of weak interactions, it may not be too objectionable to assume the gauge symmetry of strong interaction as a starting point. Therefore, we take as our starting assumptions

the following:

- i) the local symmetry of electroweak and strong interactions at high energies is given by $SU(3)_c \times U(1)_Y \times G'$;
- ii) $SU(3)_c$ is vector-like;
- iii) and that the set of Y-charges of the fermions is irreducible.

The first two assumptions are self-explanatory but the third needs to be explained. By *irreducible* set of Y-charges, we mean that if any one or two Y-charges are such that a gauge invariant mass is allowed for the fermion or the fermion pair, then we forbid such an assignment. The physical argument behind this assumption is that the value of the gauge invariant mass can be arbitrarily large compared to the scale of the gauge symmetry (i.e. $U(1)_Y \times G'$) and the fermions then will not be part of the low energy spectrum.

We will show that the above assumptions combined with the requirement that the gauge group be anomaly-free leads to the following conclusions[17]:

- (a) The minimal number of fermions that leads to an anomaly-free theory is 15, which is precisely the number of fermions in the one-generation standard model;
- (b) The maximal allowed simple G' is $SU(2)$ which must be parity-violating;

This set of assumptions appears more economical than those made in the usual construction of the standard model and the fact that one can reproduce two of the key ingredients of the one generation standard model i.e. the number of fermions, the weak gauge group is quite intriguing. Supplemented

by an extra assumption that QED is vector-like, enables one reproduce the quantum numbers of the basic fermions of the model , thereby , explaining one of the major mysteries of theoretical physics i.e. why are the observed electric charges quantized ?

In order to prove the above assertion, let us say that the number of fermions is N and is divided into two groups called quarks and leptons, the quarks being defined as triplets or anti-triplets under color and leptons being singlets. The assumed vector nature of QCD requires the number of triplets and antitriplets to be the same; let this number be equal to $Q/2$ where Q is hence an even number. Denoting the number of leptons by L , one has $N = 3Q + L$. Let the leptons and quarks have Y-charges y_i ($1 \leq i \leq L$) and z_j ($1 \leq j \leq Q$) respectively. The three anomaly constraints arising from $U(1)[Gravity]^2$, $U(1)[SU(3)_c]^2$ and $[U(1)_Y]^3$ lead to the following equations:

$$\sum_1^L y_i = 0; \tag{2a}$$

$$\sum_1^Q z_j = 0; \tag{2b}$$

$$\sum_1^L y_i^3 + 3 \sum_1^Q z_j^3 = 0; \tag{3}$$

We will be interested in finding the smallest N for which the y_i and z_j will satisfy Eqs.(1)-(3). The assumption of irreducibility implies that $L \geq 3$. Further since Q is even it can be 2, 4, 6 etc. If $Q = 2$, by Eq. (2b) $z_1 = -z_2$ so that the two quarks are allowed to acquire gauge invariant mass. (One

might wonder about invariance under G' ; but the only acceptable G' in this case is $U(1)$ and anomaly freedom implies that its charges must also be equal and opposite.) Thus we expect this quark pair then to decouple from the low energy spectrum. So, the smallest value of Q is 4 leading to $N = 15$. This proves the first assertion above. In the rest of the paper, we will denote the quark hypercharges by (z_1, z_2, z_3, z_4) and the leptonic ones by (y_1, y_2, y_3) .

Let us now show that the maximal group G' is $SU(2)$. First we show that $G' \neq SU(3)$. Since this $SU(3)$ must be orthogonal to color $SU(3)_c$, the three leptons must be a triplet under it in which case, Eq.(2a) implies that $y_i = 0$ and that there is no way to satisfy the anomalies arising from the G' group. This leaves as the only possible simple group $G'=SU(2)$.

As a brief digression, suppose G' were not simple but merely a $U(1)$. It was shown in ref.17 that the extra anomaly constraints associated with it imply that it is vectorlike . To see this, take the linear combinations of the two $U(1)$ charges to define two new $U(1)$'s and call their charges X and Y respectively. By means of an appropriate choice we can make it vanish for one of the leptons. Let us denote the X -charges of quarks to be x_a where $a = 1$ to 4 and those of leptons to be $x_5, x_6, 0$. The mixed $U(1)[Gravity]$ and $U(1)[SU(3)]^2$ anomalies then imply :

$$x_1 + x_2 + x_3 + x_4 = 0; \tag{4a}$$

$$x_5 = -x_6; \tag{4b}$$

$$x_1^3 + x_2^3 + x_3^3 + x_4^3 = 0; \quad (4c)$$

Again as before if we choose the X charges also to be rational, then Eq.(4), combined with the Fermat's last theorem will imply that $x_1 = -x_2$ and $x_3 = -x_4$ i.e. $U(1)_X$ is parity conserving.

The next question then arises : how do the quarks and leptons transform under this $SU(2)$ group ? Consistent with our assumptions, there can be at most one doublet among the quarks otherwise Eq.(2) will imply that the Y-charges for the quarks become reducible and the quarks will acquire an invariant mass and decouple from the low energy spectrum. The $G' \equiv SU(2)$ anomaly freedom then implies that there has to be one lepton doublet. There are now five Y-charges which describe the quark-lepton system (using $z_1 = z_2$ and $y_1 = y_2$ since they are members of the $SU(2)$ doublet). Now we[17] further demand that electromagnetism is vector-like subsequent to spontaneous symmetry breaking . We will show below that this together with eq.3 leads to a complete determination of the Y-charges and electric charge quantization. To establish this result, first note that the generator of the final unbroken $U(1)$ can be written in general as:

$$Q_e = I_{3L} + \eta Y; \quad (5)$$

The constraints of vector-like Q_e are:

$$-\frac{1}{2} + \eta y_1 = 2\eta y_1; \quad (6a)$$

$$+\frac{1}{2} + \eta z_1 = -\eta z_3; \quad (6b)$$

$$-\frac{1}{2} + \eta z_1 = +\eta(2z_1 + z_3); \quad (6c)$$

Equation (6b) and (6c) are actually equivalent; taken together, eqs.(6) imply that $\eta = -1/2y_1$ and $y_1 = z_1 + z_3$. Putting these relations in eq.(3), we get $y_1 = -3z_1$ and $z_3 = -4z_1$ as one solution that we use below. The other solutions of the cubic anomaly equation (3) are eliminated by the requirement of irreducibility. If we now rewrite Y as $Y' = 2\eta Y$, and call Y' as the standard model Y, then the electric charge formula becomes

$$Q_e = I_{3L} + \frac{Y}{2}. \quad (7)$$

The values of the new Y are precisely those of the standard model. Note also that in deriving the above electric charge formula nowhere have we used any assumption about the Higgs structure of the theory nor have we made use of the freedom from $SU(2)$ anomalies in this derivation. In this sense, this is slightly different from the discussion in ref.13.

IV. Effect of non-zero neutrino mass:

As is well known, the standard model leads to vanishing mass for the neutrinos due to the absence of the right-handed neutrino in the theory. There are several reasons to include the right-handed neutrino into the theory: one

is the aesthetic reason of restoring quark-lepton symmetry to weak interactions as discussed in the introduction; a second reason (of course much more compelling) is the accumulation of many indications for the existence of a non-zero neutrino mass such as solar neutrino puzzle, a hot component to the dark matter profile of the universe etc. The simplest way to understand the neutrino mass is to include the right-handed neutrino. In this section, we will discuss the impact of including the right-handed neutrino on the anomaly discussion of the previous section.

In the presence of ν_R , the minimal number of chiral fermions becomes sixteen and the Y-set becomes (z_1, z_2, z_3, z_4) in the quark sector and (y_1, y_2, y_3, y_4) in the lepton sector. Let us first discuss the question of possible choice for the gauge group G' . The anomaly and irreducibility (or decoupling) requirements clearly rule out both $SU(4)$ and $SU(3)$ as possible candidates. The only allowed ones are $SU(2)$ and $SU(2) \times SU(2)$. Let us focus on the case of $SU(2)$ [13,14]; the case of $SU(2) \times SU(2)$ is straightforward[14], the only modification being that to ensure vector-like electric charges one must use the appropriate charge formula that reads $Q = I_{3L} + I_{3R} + \eta Y$ instead of eq.5.

Note that the constraint equations in this case are the same as in eq. 2 and 3 except that the equation involving y's includes the y_4 in the sum over the y's. Due to the $SU(2)$ assignment, we have as before $y_1 = y_2$ and $z_1 = z_2$. Using the electric charge formula in eq. 5 and using the vector-like condition for electric charge and the cubic anomaly equation, one finds the following solutions for the hypercharges: $y_1 = -3z_1$; $y_3 = \frac{1}{2\eta} -$

$y_1; y_4 = -\frac{1}{2\eta} - y_1; z_3 = \frac{1}{2\eta} - z_1; z_4 = -\frac{1}{2\eta} - z_1$. All anomaly constraints and constraints from the requirement of vector-like QED are automatically satisfied by the above relations and there is no way to rescale the hypercharge coupling to fix the hypercharge quantum numbers uniquely as we did in the case of the standard model. The electric charge quantization, therefore, does not follow in this case. The reason is traceable to the fact that now the $B-L$ exists as a hidden gaugeable symmetry[14, 18] in this model. It was suggested by Babu and this author[14] that the way to restore charge quantization is to get rid of this hidden symmetry which amounts to having neutrino as a Majorana particle. This can be explicitly seen from the above relation involving the y 's and z 's as follows. The only gauge invariant Majorana mass possible is for the ν_R and that requires that $z_4 = 0$; this enables all the Y quantum numbers to be expressible in terms of $\frac{1}{\eta}$ and as in the previous section redefining Y in the same way leads to determination of all Y quantum numbers as well as to electric charge quantization.

V. Grand unification way to understand the standard model:

Here, we comment briefly on how far Grand unified theories go and where they fall short in providing an answer to the same questions discussed above. Grand unified theories are attractive for many reasons[17]; among the reasons for advocating the grand unified theories is one that says that it provides an answer to the long standing question of charge quantization. One might also think that these theories may also provide an understanding of the standard model since they generate at low energies the standard model as a consequence of symmetry breaking. I argue in this section that both

of these beliefs are not true. To see the point, let us remind ourselves that in constructing any typical GUT theory one makes a series of assumptions. We find that it is these assumptions which stand in the way of providing any fundamental understanding of the above questions. First, one assumes that the fundamental constituent fermions (i.e. the quarks and the leptons) belong to a specific representation of the gauge group : e.g. for SU(5), it is the anomaly free combination $\bar{\mathbf{5}}+\mathbf{10}$; in the case of SU(6), it is $2\bar{\mathbf{6}}+\mathbf{15}$ etc. The question then is what is reason behind this choice. Or to put it another way, many alternative theories could be constructed by choosing different anomaly free combinations. Secondly, even given the first assumption, one has to additionally assume specific Higgs representation for symmetry breaking in order to get the correct charges for the quarks and leptons and very easily alternative set of Higgs multiplets could be chosen so as to yield a different charge assignment for the fermions. For example,if SU(6) were the GUT group with the usual $\bar{\mathbf{6}}+\mathbf{15}$ assignment of fermions breaking the GUT symmetry by $\mathbf{6}$ -Higgs gives the correct charge assinnment of fermions whereas breaking it by $\mathbf{15}$ does not. Thus no real understanding of the electric charges emerges from the grand unified theories. For some other examples, see ref.19. It is worth pointing out that SU(5) and SO(10) appear to have some very interesting properties in this regard i.e. in the SU(5), always the correct charge assinnment emerges and for SO(10) also, I have checked that Higgs multiplets upto $\mathbf{560}$ give the correct assignment of charges once one breaks the GUT group down to the standard model. In this sense the string theories provide a very welcome relief in the sense that they not only predetermine

the fermion assignment but also the Higgs multiplets for symmetry breaking . The problem there is of course the well known one of non-uniqueness of the gauge group etc due to non-uniqueness of the vacuum.

VI. Conclusion:

In conclusion, a possible way to derive the standard model for one generation of fermions from a more economical set of assumptions than is commonly used, is presented and it is shown, how in this framework one can understand such mysteries of nature as the quantization of electric charge without necessarily invoking magnetic monopoles or the idea of grand unification. In this derivation, the new concept of an irreducible Y -set has been used. A single family standard model is nothing but a fifteen entry irreducible Y -set. This concept of irreducible Y -set is quite interesting and may play a role in understanding the inter-relation between generations. Applying the same considerations in the presence of a non-vanishing neutrino mass, it is shown how the possible Majorana nature of the neutrino may have a deeper physical meaning connected with electric charge quantization.

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